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Quantum Field-Theory Models on Fractal Spacetime

I. Introduction and Overview

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Abstract. The present work explores the possibility of giving a non-perturbative definition of the quantum field-theory models in non-integer dimensions, which have been previously studied by Wilson and others using analytic continuation of dimension in perturbation integrals. The method employed here is to base the models on fractal point-sets of non-integer Hausdorff-Besicovitch dimension. Two types of scalar-field models are considered: the one has its propagator (= covariance operator kernel) given by a proper-time or heat-kernel representation and the other has a hierarchical propagator. The fractal lattice version of the proper-time propagator is shown to be reflection-positive. The hierarchical models are introduced and their properties discussed on an informal basis.

1. Introduction

In a classic 1973 paper, "Quantum Field-Theory Models in Less Than 4 Dimensions," Wilson studied the scalar interaction ϕ^4 and Fermi-type $(G\bar{\psi}\psi)^2$ interaction for spacetime dimension between 2 and 4 [45]. His method was perturbative (although in some cases infinite classes of diagrams were summed within a $1/N$ expansion) and the integrals associated to the Feynman-graphs were extended to noninteger d by the analytical continuation procedure introduced earlier as a regularization method for gauge theories [10, 33]. Since that time the question of what non-perturbative significance might be given to these models – if any – has remained open. However, more recently Gefen, Aharony, Mandelbrot and collaborators have made a relevant investigation of the possibility of achieving statistical-mechanical spin models, with the critical properties predicted by the ε-expansion method, by employing fractal lattices [25–30]. In this paper the same method is exploited to give a non-perturbative definition of quantum field-

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