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Quantum K-Systems

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Abstract. We generalize the classical notion of a K-system to a non-commutative dynamical system by requiring that an invariantly defined memory loss be 100%. We give some examples of quantum K-systems and show that they cannot contain any quasi-periodic subsystem.

1. Introduction

There seems to be general agreement [1–4] that classical K-systems exhibit those mixing and chaotic properties which are necessary for the foundation of statistical mechanics. Classically they can be characterized by the existence of a subalgebra $\mathscr{A} \subset \mathscr{M} =$ the algebra of observables with

(i) $\sigma^n \mathscr{A} \supset \mathscr{A} \forall n \in \mathbf{Z}^+$,

(ii) $\bigvee_{n \geqq 0} \sigma^n \mathscr{A} = \mathscr{M}$,

(iii) $\bigwedge_{n \geqq 0} \sigma^{-n} \mathscr{A} = c\mathbf{1}$.

Here σ is the time evolution and $\vee$ and $\wedge$ mean union and intersection of algebras. These conditions are met in particular if there exists a generating subalgebra $\mathscr{A}_0 \subset \mathscr{M}$ with $\bigvee_{-\infty < n < \infty} \sigma^n \mathscr{A}_0 = \mathscr{M}$, $\bigwedge_{n=1}^{\infty} \bigvee_{j=1}^{\infty} \sigma^{-n-j} \mathscr{A}_0 = c\mathbf{1}$. The difficulties of generalizing this for non-commutative algebras $\mathscr{M}$ comes from the fact that then even two finite-dimensional isomorphic subalgebras may generate algebraically an infinite-dimensional $\mathscr{M}$. For instance, if x and p satisfy $[x, p] = i$ and χ is a characteristic function of $[-1, 1]$ and $\sigma:(x, p) \to (p, -x)$, then $\mathscr{A}_0 = (\chi(x), 1 - \chi(x))$ and $\sigma\mathscr{A}_0$ generate the algebra $W = l_\alpha \otimes M_2$ and $\mathscr{A}_0 \wedge \sigma\mathscr{A}_0 = c\mathbf{1}$. Nevertheless, Emch [2] has proposed a notion of a non-commutative K-system and an associated dynamical entropy starting with the algebraic characterization given at the beginning (see also [3, 4]). We have recently [5] given an alternative definition of the dynamical entropy of a non-commutative system and we propose a corresponding notion of a quantum K-system. We start with the classically