Commun. Math. Phys. 125, 515–526 (1989)

Communications in
Mathematical
Physics

Soft Breaking of Gauge Invariance in Regularized Quantum Electrodynamics

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Abstract. An alternative proof of the Ward-Takahashi identity for perturbative quantum electrodynamics is given which makes no use of a gauge invariant regularization such as the Pauli-Villars loop subtraction or dimensional regularization. Instead, it is shown, in the presence of an arbitrary high momentum cutoff Λ, that the exact W-T identity holds with an error of $O(1)$ $\Lambda^{-\varepsilon}$ with $0 < \varepsilon < 1$. The proof involves a perturbative analysis of the Euclidean functional integral for QED by the tree expansion method.

1. Introduction

A distinguishing feature of QED, and one which leads to considerable difficulties, is its gauge invariant character. The action for classical electrodynamics has a large symmetry group, known as the $U(1)$ gauge group, and this leads to the well-known problem of a functional integral which is constant along orbits of infinite volume, and is hence infinite.

The standard method for handling this problem is known as gauge-fixing. An extra term is added to the action, which has the effect of introducing decay along the gauge orbits. The original gauge invariance is broken, but is replaced by a weaker invariance characterized by a functional equation known as the Ward-Takahashi identity. If the W-T identity can be demonstrated in the renormalized perturbation theory, various important questions can be answered. For example, it can be shown that the theory is perturbatively unitary and that the S-matrix is independent of the choices involved in gauge-fixing [7].

Thus, an essential part of the problem of perturbative renormalization of QED is to demonstrate the W-T identity appropriate to the choice of gauge-fixing. A formal derivation of the correct W-T identity can be obtained by a change of variables in the unrenormalized functional integral provided the action has no non-invariant terms other than the gauge-fixing term. The problem is to prove that the identity one arrives at in this way actually holds after renormalization.

* Research supported by the Natural Sciences and Engineering Research Council