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Monopoles and Baker Functions

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Abstract. The work in this paper pertains to the solutions of Nahm's equations, which arise in the Atiyah–Drinfield–Hitchin–Manin–Nahm construction of solutions to the Bogomol'nyi equations for static monopoles. This paper provides an explicit construction of the solution of Nahm's equations which satisfy regularity and reality conditions. The Lax form of Nahm's equations is reduced to a standard eigenvalue problem by a special gauge transformation. These equations may then be solved by the method of Baker–Krichever. This leads to a compact representation of the solutions of Nahm's equations. The regularity condition is shown to be related to the monodromy of the gauge reduced linear operator. Hitchin showed that the solutions of Nahm's equations can be characterized by an algebraic curve and some data on that curve. Here, this characterization reduces to a transcendental equation involving certain loop integrals of a meromorphic differential. Donaldson coordinatized the moduli space of k-monopoles by a class of rational maps from the Riemann sphere to itself. The data of a Baker function is equivalent to this map. This method gives an "apriori" construction of the (known) two monopole solutions. We also give a generalization of the two monopole solution to a class of elliptic solutions of arbitrary charge. These solutions correspond to reducible curves with elliptic components and the associated Donaldson rational function has a simple partial fraction expansion.

Introduction

The work in this paper pertains to the solutions of the $SU(2)$-Bogomol'nyi [1] equations. These solutions are called monopoles and have been the subject of extensive analysis by Nahm [2], Atiyah and Hitchin [3], and Donaldson

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