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The Asymptotics of the Ray-Singer Analytic Torsion Associated with High Powers of a Positive Line Bundle

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Abstract. The purpose of this paper is to establish an asymptotic fomula for the Ray-Singer analytic torsion associated with increasing powers of a given positive line bundle.

Let M be a compact complex manifold, equipped with a smooth Hermitian metric. Let ξ be a holomorphic Hermitian vector bundle, and let μ be a positive Hermitian line bundle. For $p \in N$, let E_p be a holomorphic Hermitian flat vector bundle on M. By the Kodaira vanishing theorem, for p large enough, the sheaf $\mathcal{O}_M(\mu^{\otimes p} \otimes \xi \otimes E_p)$ has cohomology only in degree zero. We first equip μ with a metric whose curvature is positive. Let τ_p be the Ray-Singer analytic torsion [RS] of the Dolbeault complex on $\mu^{\otimes p} \otimes \xi \otimes E_p$. In this paper, we establish an asymptotic formula for $\mathrm{Log}(\tau_p)$ as $p \to +\infty$. To prove this formula, we essentially use results of Bismut [B1], where the asymptotic Morse inequalities of Demailly [D1] were established by using a heat equation method. We also establish an asymptotic bound for $\mathrm{Log}(\tau_p)$ when the metric on μ is arbitrary.

This work was partly motivated by a lecture given in February 1988 by Y. Miyaoka on his current research on analytic torsion. Our asymptotic results may have useful applications in the theory of determinant bundles of direct images and their Quillen metrics, along the lines of Quillen [Q] and Bismut-Gillet-Soulé [BGS2]. In view of a result of Bost [Bo], which relates Quillen metrics to metrics introduced in Faltings [F], our asymptotic formula refines an asymptotic result of Faltings [F, Theorem 2]. It has been recently used by Gillet and Soulé in [GS] to establish ampleness results for arithmetic manifolds, and also by Vojta [V].

Our paper is organized as follows. In Sects. a)–e), we assume that the metric on the positive line bundle μ has positive curvature. In a) we give our main assumptions and notations. In b) we establish a key estimate for the lowest eigenvalue of the Hodge Laplacian associated with the Hermitian vector bundle $\mu^{\otimes p} \otimes \xi \otimes E_p$. In c),

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