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BRST Cohomology of the Super-Virasoro Algebras

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Abstract. We study the superextension of the semi-infinite cohomology theory of the Virasoro Algebra. In particular, we examine the BRST complex with coefficients in the Fock Space of the RNS superstring. We prove a theorem of vanishing cohomology, and establish the unitary equivalence between a positive definite transversal space, a physical subspace and the zeroth cohomology group. The cohomology of a subcomplex is identified as the covariant equivalent of the well-known GSO subspace. An exceptional case to the vanishing theorem is discussed.

0. Introduction

The BRST approach has long been known to be an effective method for studying quantization of string theories. It was first applied to the Virasoro algebra of the bosonic string by Kato and Ogawa [11]. Based on a vanishing theorem, unitary equivalence between the BRST cohomology groups and the physical spaces known to physicists was proven by Frenkel, Garland and Zuckerman (FGZ) [5, 17]. They have also provided a conceptual proof of the no-ghost theorem. Several authors have recently studied the BRST quantization of the Ramond–Neveu–Schwarz (RNS) model [13, 15]. In their work, a BRST differential operator was defined and shown to be nilpotent at the critical dimension of spacetime $D = 10$ together with an appropriate normal ordering. An extension of the GSO (Gilozzi, Scherk, Olive)-projection was also proposed.

In this paper, we apply some of the ideas introduced in [5] to the Super-Virasoro algebras. Using some standard techniques in homological algebra, we prove a vanishing theorem. Formal characters and signatures of the cohomology groups are expressed in terms of modular functions. We show that the canonical hermitian forms on the BRST complexes naturally lead to ones on the relative subcomplexes and induce an (positive definite) inner product on the physical spaces. We define

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