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Inverse Spectral Problem for the Schrödinger Equation with Periodic Vector Potential

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Abstract. For the Schrödinger operator with periodic magnetic (vector) and electric (scalar) potentials a new system of spectral invariants is found. These invariants are enough to prove the rigidity of isospectral deformations in the class of generic even and real analytic magnetic and electric potentials.

1. Introduction

Let L be a lattice in $\mathbf{R}^2$ with a basis d_1, d_2, i.e. any $d \in L$ can be represented in the form

$$d = md_1 + nd_2, \quad m, n \in \mathbf{Z}.$$

Denote by L' the dual lattice, i.e. $L' = \{\delta = m\delta_1 + n\delta_2\}$, where $\delta_k \cdot d_k = 1$, $k = 1,2$, $\delta_i \cdot d_k = 0$ for $i \neq k$, $\delta \cdot d$ is the scalar product in $\mathbf{R}^2$. Let $A_k(x_1, x_2), k = 1,2, V(x_1, x_2)$ be real-valued C^∞ functions periodic with respect to the lattice L. Consider the Schrödinger equation describing the election in an electromagnetic field (see, for example [1])

$$\left(i\frac{\partial}{\partial x_1} + A_1(x)\right)^2 \psi + \left(i\frac{\partial}{\partial x_2} + A_2(x)\right)^2 \psi + V(x)\psi(x) = \lambda\psi(x), \tag{1.1}$$

where $\vec{A}(x) = (A_1(x), A_2(x))$ is the vector potential and $V(x)$ is the scalar (electric) potential. Without loss of generality we shall assume that

$$\operatorname{div} \vec{A} = \frac{\partial A_1}{\partial x_1} + \frac{\partial A_2}{\partial x_2} = 0. \tag{1.2}$$

Since $\vec{A}(x)$ is periodic we have that the magnetic field

$$B(x) = \operatorname{curl} \vec{A} = \frac{\partial A_1}{\partial x_2} - \frac{\partial A_2}{\partial x_1} \tag{1.3}$$

is also periodic and moreover

$$\iint\limits_{\mathbf{R}^2/L} B(x_1, x_2)\,dx_1\,dx_2 = 0. \tag{1.4}$$