

Superselection Sectors with Braid Group Statistics and Exchange Algebras

I. General Theory

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Abstract. The theory of superselection sectors is generalized to situations in which normal statistics has to be replaced by braid group statistics. The essential role of the positive Markov trace of algebraic quantum field theory for this analysis is explained, and the relation to exchange algebras is established.

1. Introduction

Superselection sectors in 4 dimensional quantum field theories are classified by the equivalence classes of irreducible representations of some compact group, the group of internal symmetries. All models seem to have this property, and recently, Doplicher and Roberts succeeded in deriving the existence of such a group from first principles [1]. Their treatment is based on the theory of superselection sectors [2] which has been developed in the framework of algebraic quantum field theory [3]. The basic result of the theory of superselection sectors is the intrinsic definition of statistics. There is, associated with each sector, an – up to equivalence – unique representation of the permutation group which describes the statistics of multi-particle states. In principle, the theory can be applied also to models in lower dimensional space time, however, there statistics has to be described, in general, by a representation of the braid group.¹

In view of the recent progress in the analysis of representations of the braid group [4] it seems to be worthwhile to analyze those representations which occur in quantum field theory more closely. On the other hand, nowadays a lot of models are known, especially conformally covariant field theories in two dimensions, which exhibit a rich structure of superselection sectors which does not seem to fit the representation theory of some group. Actually, representations of the braid group

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¹ This fact seems to be well known to the experts. We thank D. Buchholz, S. Doplicher, J. Fröhlich, J. Roberts, and R. Tscheuschner for helpful discussions on this point