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Factorisation of Energy Dependent Schrödinger Operators: Miura Maps and Modified Systems

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Abstract. We consider the energy dependent Schrödinger operator $\mathbb{L}=\sum_{i=0}^{N}\lambda^i(\varepsilon_i\partial^2+u_i)$, which we have previously shown to be associated with multi-Hamiltonian structures [2]. In this paper we use an unusual form of the Lax approach to derive by a *single construction* the time evolutions of the eigenfunctions of $\mathbb{L}$, the associated Hamiltonian operators and the Hamiltonian functionals. We then generalise the well known factorisation of standard Lax operators to the case of energy-dependent operators. The simple product of linear factors is replaced by a λ-dependent quadratic form. We thus generalise the resulting construction of Miura maps and modified equations. We show that for some of our systems there exists a sequence of N such modifications, the r^{th} modification possessing $(N-r+1)$ Hamiltonian structures.

1. Introduction

In a number of recent papers [1–4] we discussed the two generalised Schrödinger equations:

$$L_1\psi\equiv\left(\sum_{0}^{N-1}\lambda^i(\varepsilon_i\partial^2+u_i)\right)\psi=\lambda^N\psi\,, \tag{1.1a}$$

$$L_2\psi\equiv\left(\partial^2+\sum_{1}^{N}u_i\lambda^i\right)\psi=a^2\psi\,, \tag{1.1b}$$

where a is a constant. We have shown that the isospectral flows of each of these spectral problems possess $(N+1)$ compatible Hamiltonian structures $\mathbf{B}_0,\ldots,\mathbf{B}_N$. When $N=1$ these spectral problems give rise respectively to the KdV and Harry

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