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The BRST Cohomology of the NSR String: Vanishing and "No-Ghost" Theorems

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Abstract. We investigate the BRST cohomology of the NSR string. We prove vanishing theorems for the full and relative (sub)complexes generalizing the work of Frenkel, Garland and Zuckerman for the bosonic string. Using these results we give simple proofs of the "no-ghost" theorems for both sectors.

1. Introduction

In [1] Frenkel, Garland, and Zuckerman computed the BRST cohomology of the open bosonic string after identifying it with a particular semi-infinite cohomology for which they had proven a key vanishing theorem; that is, that the BRST cohomology is zero except at zero ghost number. Their vanishing theorem did not just apply to that particular representation of the Virasoro algebra but to a large class of graded Lie algebras (including Kac-Moody algebras) and to a large class of their representations. In the present paper we extend their result to the representations of the super-Virasoro algebras appearing in the NSR string. The theorem admits some generalization which, lacking the string theoretic relevance of this special case, will be presented elsewhere. This way we can devote ourselves to the cases of current physical interest and therefore give a clear presentation of the method without needless generalizations.

The proof uses the algebraic machinery of spectral sequences. Since this lies somewhat outside the physicist's bag of tricks we thought it would be convenient to devote the next section to take a brief look at this powerful gadget. That section also serves to clarify the notation and the concepts concerning differential complexes that we use in this paper. We define the notion of a filtered complex and quote the main theorem concerning the spectral sequence associated to it. A very important special case of a filtered complex, and one for which we will find ample use, is the double complex. We will see that there are two canonical filtrations associated to a

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