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Non-Holonomicity of the *S* Matrix and Green Functions in Quantum Field Theory: A Direct Algebraic Proof in Some Simple Situations

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Abstract. A criterion of non-holonomicity of the S matrix and Green functions in some basic simple situations of relativistic quantum field theory, previously established in an indirect way or with supplementary assumptions, is reobtained from unitarity equations by a direct and general algebraic argument.

1. Introduction

Holonomicity in the sense of M. Sato [SKK, KK] is an important notion in the analysis of singularities of distributions (or hyperfunctions). Its meaning in situations of interest in this note is recalled below. It was conjectured in [S] that the momentum-space S matrix and Green functions of relativistic quantum field theory should satisfy holonomicity properties at their Landau singularities. As a matter of fact, holonomicity is satisfied in some simple well-known situations, where singularities are poles, logarithms or, at 2-particle thresholds, square-roots, and also [KS1] for a particular class of singularities which includes previous ones. On the other hand, Feynman integrals are always holonomic (see [KK] and references therein). However, the further analysis from various viewpoints [BI1,2, BP, KS2] (perturbative or non-perturbative field theory, S-matrix theory) indicates that the S matrix and Green functions are probably non-holonomic in general and leads one to consider formulations of the idea of [S] involving infinite convergent expansions in terms of holonomic contributions. For related investigations and results in particular in constructive field theory, see [I1, IM, I2].

Actual proofs of non-holonomicity have been given in [BI1, BP] in some basic simple situations. The purpose of this note is to present a new more direct proof, providing a better understanding of mechanisms that generate non-holonomicity in these situations: at the 2-particle threshold $s=4\mu^2$ itself if the dimension d of space-time is odd and, more generally, in a simplified theory with no subchannel interaction, at the m-particle threshold $s=(m\mu)^2$ in a $m\to m$ process if

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