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YM_2: Continuum Expectations, Lattice Convergence, and Lassos

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Abstract. The two dimensional Yang-Mills theory (YM_2) is analyzed in both the continuum and the lattice. In the complete axial gauge the continuum theory may be defined in terms of a Lie algebra valued white noise, and parallel translation may be defined by stochastic differential equations. This machinery is used to compute the expectations of gauge invariant functions of the parallel translation operators along a collection of curves $\mathscr{C}$. The expectation values are expressed as finite dimensional integrals with densities that are products of the heat kernel on the structure group. The time parameters of the heat kernels are determined by the areas enclosed by the collection $\mathscr{C}$, and the arguments are determined by the crossing topologies of the curves in $\mathscr{C}$. The expectations for the Wilson lattice models have a similar structure, and from this it follows that in the limit of small lattice spacing the lattice expectations converge to the continuum expectations. It is also shown that the lasso variables advocated by L. Gross [36] exist and are sufficient to generate all the measurable functions on the YM_2-measure space.

1. Introduction

The informal expression for the Yang-Mills' measure is:

$$\mu(dA) = Z^{-1} \exp\left[\frac{1}{2g_0^2} \int_{\mathbf{R}^d} \sum_{i<j} \operatorname{trace}_\rho (F_{ij}^A(x)^2) dx\right] \mathscr{D}A, \tag{1.1}$$

where A runs over a space of connection forms ($\mathscr{A}$) *on the trivial unitary vector bundle* $C^N \times R^d$, $F^A = dA + A \wedge A$ is the curvature of A, $\mathscr{D}A = \prod_{i=1}^{d} \prod_{x \in \mathbf{R}^d} d(A_i(x))$ is "infinite dimensional Lebesgue measure" on $\mathscr{A}$, g_0^2 is a positive "coupling" constant, and Z is a normalization constant which makes μ a probability measure. The connection forms are restricted to take values in the Lie algebra $\mathscr{G}$ of the structure (or gauge) group G—a subgroup of $U(N)$. The trace is taken with respect to some representation ρ of G.

It is well known that the expression (1.1) is ill defined, see Gross [35]. Despite the many technical problems, when $d = 2$ it is possible to define a "gauge fixed" version of μ as mean zero Gaussian measure on a space of generalized connection 1-forms,