

The Operator Algebra of Orbifold Models

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Abstract. We analyze the chiral properties of (orbifold) conformal field theories which are obtained from a given conformal field theory by modding out by a finite symmetry group. For a class of orbifolds, we derive the fusion rules by studying the modular transformation properties of the one-loop characters. The results are illustrated with explicit calculations of toroidal and $c = 1$ models.

1. Introduction

The classification of conformal invariant field theories has up to now only been exhaustive for $c < 1$ [1–4]. This is due to the particular properties of the Virasoro algebra for these values of the central charge. It only has a finite set of unitary representations and only for these minimal models the Virasoro algebra gives enough information – in the form of null states – to determine the possible interactions. It has long been evident that extensions of the conformal symmetry are needed in order to give equally exhaustive results in the classification for $c \geq 1$. Partial results in this direction have been obtained, mainly in the context of superconformal and affine Lie algebras.

There has recently been much interest in rational conformal field theories (RCFT's) [5–13]. These theories are characterized by the fact that their correlations are given by a finite sum of holomorphic times anti-holomorphic functions of the moduli of punctured surfaces. This property of the RCFT's can be used to derive powerful constraints on the operator content and operator product relations in these models. In this respect they form a natural generalization of the minimal theories and it is clearly an important problem to try and find a complete description of all possible RCFT's. From this viewpoint, it would be very interesting to devise operations that act on the space of RCFT's and generate new theories from a given one. As an example one can think of the construction of non-diagonal modular invariant combinations of characters, coset models [14], etc. Another operation of that kind is the concept of an orbifold.