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Non-Archimedean Strings and Bruhat-Tits Trees

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Abstract. It is shown that the Bruhat-Tits tree for the p-adic linear group $GL(2)$ is a natural non-archimedean analog of the open string world sheet. The boundary of the tree can be identified with the field of p-adic numbers. We construct a "lattice" quantum field theory on the Bruhat-Tits tree with a simple local lagrangian and show that it leads to the Freund-Olson amplitudes for emission processes of the particle states from the boundary.

1. Introduction

In the early days of string theory the string world sheet was thought of as being imbedded into a D-dimesnional space-time, with the string action given by the area swept by the world sheet (the Nambu-Goto action [1]). In the course of further development, the approach due to Polyakov [2] proved more fruitful for quantization purposes; the string world sheet was now looked at as an abstract surface with a metric living on it. More precisely, the prescription was to consider a certain two-dimensional theory with its fields taking values in a D-dimensional space, and functionally integrate over the fields as well as the metrics. In critical dimension ($D=26$) the world sheet can be thought of as an algebraic curve over the complex number field, which has allowed for diverse applications for the methods of modern algebraic geometry in perturbative calculations of string amplitudes [3]. Adopting the algebraic-geometrical point of view, it would be quite natural to go beyond the complex number field and to consider other number fields on an equal footing. The first attempts of doing so were made in recent papers [4–6].

A further extension of the notion of the bosonic string world sheet was suggested by Freund and Olson [4], who started with the fact that, as the physically sensible quantities (scattering amplitudes) are given by integrals of complex-valued functions on the world sheet, the world sheet by itself is unobservable. So they chose it to be a manifold over p-adic number field Q_p (p – a prime integer). From the conventional point of view based on the archimedean topology, non-archimedean local fields Q_p possess a number of peculiar "fractal"