Commun. Math. Phys. 123, 425–452 (1989)

Communications in
Mathematical
Physics

The Sine-Gordon Field Theory Model at $\alpha^2 = 8\pi$, the Non-Superrenormalizable Theory

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Abstract. We study the Sine-Gordon field theory at $\alpha^2 = 8\pi$. We prove that the theory is renormalizable but not superrenormalizable and we show how the perturbative renormalization procedure works in this case where the interaction is not polynomial. To go beyond the perturbative results we investigate the β-functional equation for this theory and discuss in what sense at $\alpha^2 = 8\pi$ the theory is lacking the asymptotic freedom and how it is asymptotic free for $\alpha^2 < 8\pi$ in a appropriate region of the coupling constants.

Introduction

We study the renormalizability and the asymptotic freedom of the field theory model defined by the potential

$$V(\varphi) = \lambda \int_{\Lambda} d^2x : \cos \alpha\varphi_x : = \frac{\lambda}{2} \sum_{\sigma = \pm 1} \int_{\Lambda} d^2x : e^{i\alpha\sigma\varphi_x} : , \tag{0.1}$$

where $\lambda \in R \backslash 0$, $\alpha^2 = 8\pi$, Λ is a finite volume in the euclidean $d = 2$ space-time with periodic boundary conditions. The properties of the Sine-Gordon model crucially depend on the α value:

a) $\alpha^2 \in [4\pi, 8\pi)$.

The model is superrenormalizable; there is a finite number of divergences which can be cured with field independent counterterms. Moreover as α^2 approaches 8π the number of divergences tends to infinity. It has been proven in a series of papers [1–4] that in this range of values, for λ sufficiently small, the model exists.

b) $\alpha^2 = 8\pi$

i) The model is only renormalizable. The number of divergences does not depend on the order of the perturbation theory and three types of counterterms are necessary, two of them field dependent. The renormalizability of the model has been proven only in a perturbative sense.