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Ground State(s) of the Spin-Boson Hamiltonian

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Abstract. We establish that the finite temperature KMS states of the spin-boson hamiltonian have a limit as the temperature drops to zero. Using recent advances on the one-dimensional Ising model with long range, $1/r^2$, interactions, we prove qualitative properties of the ground state(s) in the ohmic case. We show (i) the asymptotics of the critical coupling as the bare energy gap goes to zero and to infinity, (ii) a jump in the order parameter, and (iii) that the number of bosons is finite below and infinite at and above the critical coupling strength.

I. Introduction

The spin-boson Hamiltonian models a "spin" coupled to a Bose field and is given by

$$H = -\varepsilon\sigma_x + \int dk\omega(k)a^+(k)a(k) + \sqrt{\alpha/2}\,\sigma_z \int dk\lambda(k)(a^+(k) + a(k)) - h\sigma_z. \quad (1.1)$$

Here σ_x, σ_z are the Pauli spin matrices. $a(k)$, $a^+(k)$ is a scalar Bose field with commutation relation $[a(k), a^+(k')] = \delta(k-k')$. The Bose field is over $\mathbf{R}^d$, the d-dimensional Euclidean space. $\omega(k)$ is the dispersion relation of the field, $\omega(k) \geqq 0$. $\lambda(k) = \lambda(k)^*$ are the couplings.

The spin-boson system is a prototype for the interaction of a localized degree of freedom with a field. No wonder the spin-boson Hamiltonian has a rich history. The reader is referred to [1] for an excellent and up-to-date review.

We consider here only the case characteristic of most physical applications, the so-called ohmic case: the frequency distribution is linear for small ω, i.e.

$$\int dk\lambda(k)^2\delta(\omega(k) - \omega) \simeq \omega. \quad (1.2)$$

Equivalently

$$W(t) = \int dk\lambda(k)^2 e^{-\omega(k)|t|} \simeq \frac{1}{t^2} \quad (1.3)$$

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