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Classical and Quantum Conformal Field Theory

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Abstract. We define chiral vertex operators and duality matrices and review the fundamental identities they satisfy. In order to understand the meaning of these equations, and therefore of conformal field theory, we define the classical limit of a conformal field theory as a limit in which the conformal weights of all primary fields vanish. The classical limit of the equations for the duality matrices in rational field theory together with some results of category theory, suggest that (quantum) conformal field theory should be regarded as a generalization of group theory.

1. Introduction and Conclusion

Although the classification of conformal field theory is an extremely interesting problem, of importance in mathematics, statistical mechanics, and string theory, it should not be mistaken for a fundamental problem in string theory. Conformal field theories are classical solutions of the string equations of motion. In string theory the basic physical laws which lead to the string equations are far more important than the classification of the solutions to those equations. However, the meaning of these equations is far from being understood. It seems that our lack of a full understanding of the meaning of conformal field theory prevents us from finding natural generalizations. One might hope that a proper formulation of conformal field theory will lead us to the principles underlying string theory and will allow us to generalize the "on-shell" results. The classification of conformal field theories should be viewed as a step in this direction – a concrete way of thinking about the more important problem of the meaning of conformal field theory.

The classification of all conformal field theories is an enormous problem. To make it tractable, physicists have proceeded by solving the problem in stages. The

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