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Multiparametric Quantum Deformation of the General Linear Supergroup

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Abstract. In the work L. D. Faddeev and his collaborators, and subsequently V. G. Drinfeld, M. Jimbo, S. L. Woronowicz, a new class of Hopf algebras was constructed. They can be considered as one-parametric deformations of either group ring or the universal enveloping algebra of a simple algebraic group. In this paper we define and investigate a multiparametric deformation of the general linear supergroup. This is the simplest example of some general constructions described in [5, 6].

Introduction

Quantum groups were recently introduced and studied from various viewpoints in the work of Faddeev and his collaborators (cf. [3] and references therein) followed by Drinfeld [2], Jimbo [4], and Woronowicz [7].

In [5, 6] I described a class of quantum groups which are natural symmetries of non-commutative algebraic varieties defined by quadratic equations. This approach furnishes a vast supply of new quantum groups together with their representations.

This paper is devoted to the detailed and explicit study of the simplest groups that can be constructed in this way. It can be read independently of [5, 6] although it is useful to keep in mind the underlying philosophy explained in [6]. In addition, we take into account some new effects of a structural $\mathbf{Z}_2$-grading thus superizing some parts of [5, 6].

Our working definition of a quantum (super)group is that of a Hopf (super)algebra generated by the entries of a multiplicative matrix i.e., admitting a faithful finite-dimensional corepresentation. This viewpoint is close to that of [3, 7] but dual to that of [2, 4].

The paper is structured as follows. In Sect. 1 we state all essential definitions and results. Proofs and some complements are given in Sects. 2–5.

1. Notation and Results

1.1. Hopf Superalgebras and Quantum Supergroups. All our objects are defined over a ground field k of characteristic $\neq 2$ e.g., $\mathbf{R}$ or $\mathbf{C}$. An associative algebra with