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Existence, Regularity, and Asymptotic Behavior of the Solutions to the Ginzburg-Landau Equations on $\mathbb{R}^3$

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Abstract. This paper studies the solutions of the Ginzburg-Landau equations on $\mathbb{R}^3$ in the presence of an arbitrarily distributed external magnetic field. The existence and regularity of the solutions at the lowest energy level are established. The solutions found are in the Coulomb gauge. If the external field is sufficiently regular, the solutions are shown to have nice asymptotic decay properties at infinity.

1. Introduction

In the Ginzburg-Landau semi-quantum mechanical theory of superconductivity the behavior of a superconductor cooled below the transition temperature in the absence of an external magnetic field is described by the equations

$$\left.\begin{aligned} D_A^2\phi + \frac{\lambda}{2}(1-|\phi|^2)\phi = 0\,, \\ \mathrm{curl}^2\mathbf{A} + \frac{i}{2}(\phi^* D_A\phi - \phi(D_A\phi)^*) = 0\,, \end{aligned}\right\} \tag{1.1}$$

which are the equations of motion of the free energy density

$$\mathscr{E} = \frac{1}{2}|\mathrm{curl}\,\mathbf{A}|^2 + \frac{1}{2}|D_A\phi|^2 + \frac{\lambda}{8}(|\phi|^2-1)^2\,. \tag{1.2}$$

Here the complex scalar field ϕ is an order parameter so that $|\phi|^2$ gives the relative density of the superconducting condensed electron pairs, called the Cooper pairs which behave like charged bosonic particles, $\mathbf{A}$ is a gauge photon field, and $D_A\phi = \nabla\phi - i\mathbf{A}\phi$. In this model, $\lambda > 0$ is a dimensionless coupling constant with $\lambda < 1$ and $\lambda > 1$ describing type I and type II superconductors respectively, the electric field is absent, the magnetic field is determined through $\mathbf{H} = \mathrm{curl}\,\mathbf{A}$, and the ground states (or the superconducting vacua) are given by $\mathbf{A} = 0$, $\phi = e^{i\theta}$, $\theta \in \mathbb{R}^1$. The Ginzburg-Landau equations (1.1), which have been accepted as the fundamental