Commun. Math. Phys. 123, 85–93 (1989)

Communications in Mathematical Physics


# Uniform and $L^2$ Convergence in One Dimensional Stochastic Ising Models*

Richard A. Holley and Daniel W. Stroock

Department of Mathematics, University of Colorado, Boulder, Colorado 80309, USA

Department of Mathematics, Massachusetts Institute of Technology, Room 2-272, 77 Mass Ave., Cambridge, MA 02139, USA

**Abstract.** We study the rate of convergence to equilibrium of one dimensional stochastic Ising models with finite range interactions. We do *not* assume that the interactions are ferromagnetic or that the flip rates are attractive. The infinitesimal generators of these processes all have gaps between zero and the rest of their spectra. We prove that if one of these processes is observed by means of local observables, then the convergence is seen to be exponentially fast with an exponent that is any number less than the spectral gap. Moreover this exponential convergence is uniform in the initial configuration.

## 0. Introduction

The stochastic Ising model (often called the kinetic Ising model) was introduced by R. J. Glauber [RG] in 1963. The model that Glauber introduced is one dimensional and was carefully chosen so that one could explicitly compute the rate at which local observables relax to their equilibrium values. As a consequence of these explicit calculations one can see for Glauber's model that the rate at which convergence takes place when measured in the uniform norm is exactly the same as the rate when measured in the $L^2$ norm. It is the purpose of this paper to prove that, in one dimension, the same equality holds for all translation invariant, finite range interactions and all choices of flip rates that are translation invariant, have finite range, and satisfy the appropriate detailed balance condition.

By an *interaction* we mean any collection $\{J_R : R \subseteq \mathbf{Z}\} \subseteq \mathbb{R}$. We say that the interaction $\{J_R : R \subseteq \mathbf{Z}\}$ is *translation invariant* if, for every $R \subseteq Z$, $J_R = J_{R+k}$ for any $k \in \mathbf{Z}$; and we say that it has *finite range* if there is a finite number $L$ (the range) such that $J_R = 0$ whenever $\operatorname{diam}(R) > L$. We assume throughout that our interactions are translation invariant and have finite range.

Next, set $E = \{-1, 1\}^{\mathbf{Z}}$ and think of the elements $\sigma$ of $E$ as configurations on $\mathbf{Z}$ of $\pm 1$ valued spins. Thus, $\sigma_k \in \{-1, 1\}$ is the spin at site $k \in \mathbf{Z}$ of the configuration

* The authors were partially supported by N.S.F. Grants DMS-8609944 and DMS-8611487, respectively. Also, the second author acknowledges supports from DAAL 03-86-K-0171