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The Global Structure of Simple Space-Times

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Abstract. According to a standard definition of Penrose, a space-time admitting well-defined future and past null infinities $\mathscr{I}^+$ and $\mathscr{I}^-$ is asymptotically simple if it has no closed timelike curves, and all its endless null geodesics originate from $\mathscr{I}^-$ and terminate at $\mathscr{I}^+$. The global structure of such space-times has previously been successfully investigated only in the presence of additional constraints. The present paper deals with the general case. It is shown that $\mathscr{I}^+$ is diffeomorphic to the complement of a point in some contractible open 3-manifold, the strongly causal region $\mathscr{I}_0^+$ of $\mathscr{I}^+$ is diffeomorphic to $\mathbb{S}^2 \times \mathbb{R}$, and every compact connected spacelike 2-surface in $\mathscr{I}^+$ is contained in $\mathscr{I}_0^+$ and is a strong deformation retract of both $\mathscr{I}_0^+$ and $\mathscr{I}^+$. Moreover the space-time must be globally hyperbolic with Cauchy surfaces which, subject to the truth of the Poincaré conjecture, are diffeomorphic to $\mathbb{R}^3$.

1. Introduction

Consider a space-time which develops from initial data on an $\mathbb{R}^3$ Cauchy surface, and models an isolated, massive body. Suppose that the gravitational field strength is insufficient to cause collapse or to give rise to orbiting null geodesics akin to those at $r = 3m$ in Schwarzschild space-time. One may then reasonably assume that all endless null geodesics originate from a past null infinity $\mathscr{I}^-$ and escape to a future null infinity $\mathscr{I}^+$. As the space-time evolves, $\mathscr{I}^+$ is exposed to data on an increasingly large region of the Cauchy surface, and may be expected to respond by exhibiting increasingly complicated behaviour. What can be said about the general structure of $\mathscr{I}^+$, and about its global topology in particular?

In order to answer such questions, it is first necessary to specify more precisely the class of space-times to be considered. The only assumptions that will be necessary are that there are well-defined future and past null infinities $\mathscr{I}^+$ and $\mathscr{I}^-$, that all endless null geodesics originate from $\mathscr{I}^-$ and terminate at $\mathscr{I}^+$, and that there are no closed timelike curves. The existence of an $\mathbb{R}^3$ Cauchy surface can, subject to the truth of the Poincaré conjecture, be derived from these hypotheses.

According to Penrose [1] one could, on physical grounds, assume that $\mathscr{I}^+$