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Modular Covariance of Minimal Model Correlation Functions

G. Felder [1,*] and R. Silvotti [2,**]

[1] School of Mathematics, Institute for Advanced Study, Princeton, NJ 08540, USA
[2] Theoretische Physik, Universität Zürich, Schönberggasse 9, CH-8001 Zürich, Switzerland

Abstract. We prove that one-point functions of all scaling fields in minimal left-right diagonal models of conformal field theory are modular covariant. This consistency condition should allow one to extend these minimal models to Riemann surfaces of arbitrary genus.

1. Introduction

In recent times, the problem of extending two-dimensional conformal field theory [1] to higher genus Riemann surfaces has received considerable attention [2, 3]. The main reason for this interest lies in the fact that quantum corrections to classical string theories are given perturbatively in terms of correlation functions of some conformal field theory on Riemann surfaces of positive genus. On the other hand, the requirement of modular invariance of the partition function on the torus has turned out to be a simple but powerful tool to write down candidates for conformal invariant models [4, 5], giving a recipe for finding universality classes of statistical systems at the critical point.

In general, however, several important questions remain unanswered. Specifically, the partition function on the torus gives only the field content of a conformal invariant theory: Is there a consistent (crossing symmetric) operator product algebra corresponding to this field content? Can this theory consistently be extended to higher genus Riemann surfaces? Answering the first question requires solving the conformal bootstrap equations [1] and checking the consistency of the solution. These equations have been solved in some models [6, 7]. As for the second question, there is general agreement [8, 9] that a necessary and sufficient condition for a consistent conformal field theory defined on the sphere to admit an extension to higher genus is that the one-point function of all scaling fields on the

* Supported in part by NSF grant DMS 8610730. Address after March 1989: Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland
** Also: Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Italy