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Cantor Spectrum and Singular Continuity for a Hierarchical Hamiltonian*

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Abstract. We study the spectrum of the Hamiltonian H on $l_2(\mathbb{Z})$ given by $(H\psi)(n)=\psi(n+1)+\psi(n-1)+V(n)\psi(n)$ with the hierarchical (ultrametric) potential $V(2^m(2l+1))=\lambda(1-R^m)/(1-R)$, corresponding to 1-, 2-, and 3-dimensional Coulomb potentials for $0<R<1$, $R=1$ and $R>1$, respectively, in a suitably chosen valuation metric. We prove that the spectrum is a Cantor set and gaps open at the eigenvalues $e_n(1)<e_n(2)<\ldots<e_n(2^n-1)$ of the Dirichlet problem $H\psi=E\psi$, $\psi(0)=\psi(2^n)=0$, $n\geqq 1$. In the gap opening at $e_n(k)$ the integrated density of states takes on the value $k/2^n$. The spectrum is purely singular continuous for $R\geqq 1$ when the potential is unbounded, and the Lyapunov exponent γ vanishes in the spectrum. The spectrum is purely continuous for $R<1$ in $\sigma(H)\cap[-2,2]$ and $\gamma=0$ here, but one cannot exclude the presence of eigenvalues near the border of the spectrum. We also propose an explicit formula for the Green's function.

Table of Contents

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