Commun. Math. Phys. 122, 597–607 (1989)

Communications in
Mathematical
Physics

A Note on the Ising Model in High Dimensions

J. Bricmont,[1,*] H. Kesten,[2,**] J. L. Lebowitz[3,***] and R. H. Schonmann[2,****]

[1] Institut de Physique Theorique, Universite Catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium

[2] Department of Mathematics, White Hall, Cornell University, Ithaca, NY 14853, USA

[3] Department of Mathematics, Busch Campus, Rutgers University, New Brunswick, NJ 08903, USA

Abstract. We consider the d-dimensional Ising model with a nearest neighbor ferromagnetic interaction $J(d) = 1/4d$. We show that as $d \to \infty$ the $+$ phase (and the $-$ phase) approaches a product measure with density given by the mean field approximation. In particular the spontaneous magnetization converges to its mean field value. A similar result holds for the unique Gibbs measure of the system subject to an external field $h \neq 0$.

I. Introduction

There exists a variety of rigorously established relations between statistical mechanical models and their mean field approximations. For several models the following types of results have been proven:

I) The mean field critical temperature is an upper bound for the critical temperature of the model (see [F] and [G] for the Ising model and [Si] for the classical Heisenberg model).

II) Convergence of the free energy to its mean field value when the dimensionality goes to infinity and the interaction is properly normalized (see [T3] for the Ising model and [PT] for generalizations; this result was first obtained nonrigorously in [Br]).

III) The mean field value of the magnetization is an upper bound for the magnetization of the model. In particular this implies the result in (I) above (see [T1] for the Ising model and for instance [Pe, N, Sl, TH, V] for generalizations).

IV) Convergence of the critical temperature to its mean field value when the

* Part of this work was done while this author was visiting Rutgers University, supported by NSF grant DMR-86-12369 and Princeton University, support by NSF grant PHY-85-15288-A01

** Partially supported by a NSF grant to Cornell University

*** Partially supported by NSF grant DMR 86-12369

**** Supported by the U.S. Army Research Office through the Mathematical Sciences Institute of Cornell and by a NSF grant to Cornell University. This author was supported by the NSF grant DMR-86-12369 while visiting Rutgers University (when this work was started). On leave from São Paulo University