Commun. Math. Phys. 122, 293–320 (1989)

Communications in
Mathematical
Physics

Continuous Measures in One Dimension*

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Abstract. Families of unimodal maps satisfying
(1) $T_\lambda:[-1,1]\mapsto[-1,1]$ with $T(\pm 1)=-1$ and $|T_\lambda'(1)|>1$,
(2) $T_\lambda(x)$ is C^2 in x^2 and λ, and symmetric in x,
(3) $T_0(0)=0$, $T_1(0)=1$ with $\frac{d}{d\lambda}T_\lambda(0)>0$
are considered. The results of Guckenheimer (1982) are extended to show that there is a positive measure of λ for which T_λ has a finite absolutely continuous invariant measure.

The appendix contains general theorems for the existence of such measures for some markov maps of the interval.

(A) Introduction

Jakobson published a theorem in 1978 that states that for the family of unimodal maps $T_\lambda:[0,1]\mapsto[0,1]$ defined by $T_\lambda(x)=\lambda x(1-x)$, there exists a set of parameters λ of positive lebesque measure for which T_λ possesses a finite absolutely continuous invariant measure.

Since that time there have been several attempts to present a more comprehendable proof. Rychlik (1986) has produced a cleaner proof than Jakobson using similar techniques. Benedicks and Carleson (1983) use statistical methods and a widey different approach to achieve a similar result. Rees (1985) has presented a proof in the complex case.

In 1982 Guckenheimer proved that for reasonable families of unimodal maps, there is a positive measure set of parameter values for which the corresponding map has sensitivity to initial conditions (a map T has sensitivity to initial conditions if $\exists\varepsilon>0\,\forall x,\ \delta>0\,\exists y,\ |x-y|<\delta,\ \exists n,\ |T^n(x)-T^n(y)|>\varepsilon$). Specifically, he

* Supported by the U.S. Army Research Office through the Mathematical Sciences Institute of Cornell University