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Compactifications of $d = 11$ Supergravity on Kähler Manifolds*

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Abstract. We consider compactifications of eleven-dimensional supergravity to five and three spacetime dimensions, on internal spaces K_6 and $K_2 \times K_6$, where K_n denotes an n-dimensional Kähler manifold. The compactifications to five dimensions yield no surviving spacetime supersymmetries. However, we find compactifications to three dimensions on $S^2 \times K_6$ and $T^2 \times K_6$ where K_6 is Ricci-flat and Kähler (a Calabi-Yau space) with $N = 4$ supersymmetry. We also discuss the massless spectrum.

The eleven-dimensional supergravity theory attracted much attention a few years ago as a possible candidate for a unified theory of all the fundamental interactions, including gravity. These original hopes seemed not to be realized in practice, and with the growing surge of interest in superstring theory, work on eleven-dimensional supergravity was largely abandoned. Recently, however, eleven-dimensional supergravity has been making a modest comeback, as a result of developments in supersymmetric theories of higher-dimensional extended objects (p-branes) [1]. In particular, the eleven-dimensional supermembrane seems to have the best chance of being consistent at the quantum level, although at this stage the evidence is only circumstantial. It has, however, survived consistency checks which appear to fail for all the other super p-brane theories. As one might expect, the eleven-dimensional supermembrane is intimately related to eleven-dimensional supergravity, in much the same way as the ten-dimensional superstring is related to ten-dimensional supergravity. In particular, it can be embedded in an eleven-dimensional supergravity background, and the three-dimensional supermembrane action exhibits a local fermionic "Siegel"-symmetry if the background satisfies the classical equations of motion of eleven-dimensional supergravity.

In the light of the renewed interest in $d = 11$ supergravity as a possible low-energy limit of the supermembrane theory, it seems appropriate to re-examine the theory. In particular, just as superstrings emphasize the importance of two-dimensional supersymmetric theories, so supermembranes emphasize the

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