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Block Renormalization Group for Euclidean Fermions

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Abstract. Block renormalization group transformations (RGT) for lattice and continuum Euclidean Fermions in d dimensions are developed using Fermionic integrals with exponential and "δ-function" weight functions. For the free field the sequence of actions D_k generated by the RGT from D, the Dirac operator, are shown to have exponential decay; uniform in k, after rescaling to the unit lattice. It is shown that the two-point function D^{-1} admits a simple telescopic sum decomposition into fluctuation two-point functions which for the exponential weight RGT have exponential decay. Contrary to RG intuition the sequence of rescaled actions corresponding to the "δ-function" RGT do not have uniform exponential decay and we give examples of initial actions in one dimension where this phenomena occurs for the exponenential weight RGT also.

I. Introduction

Recently rigorous renormalization group methods have been successfully applied to analyze many lattice and continuum models in statistical mechanics and in the Euclidean version of quantum field theory [1–25]. A common feature of the methods applied to the analysis of field theory models in the Euclidean framework is a decomposition of the momentum space achieved by the decomposition of a free continuum propagator or in the case of lattice regularizations by block field transformations. Block renormalization group methods have been used in abelian gauge, abelian gauge–Boson and non-abelian gauge lattice models in two, three and four dimensions to show ultraviolet stability [1–7]. In this paper with a view to future applications to lattice regularized Fermionic and gauge–Fermionic models (see [26–27] for the lattice formulation) we develop the block renormalization group for lattice Fermions. Our treatment parallels the one of [1–3] for scalar and vector Bosons where the RGT is an integral transformation with a Gaussian weight function; for the δ-function weight for the lattice scalar field see [13–14]. The results for the Gaussian case are also similar except for minor changes due to differences in the canonical scaling of free Bosons and Fermions. However there is an unexpected difference in the δ-function formulation.