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A New Proof of the Propagation Theorem for N-Body Quantum Systems

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Abstract. A new proof of I. Sigal's and A. Soffer's propagation theorem is given. This theorem describes a large class of operators which are Kato-smooth with respect to an N-body Schrödinger operator.

1. Introduction

One can learn a lot about the Schrödinger operator H by studying the asymptotic behavior of certain observables in the Heisenberg picture as the time goes to infinity. There exists a number of various results on this subject, which say roughly that for large times many observables behave to some extent in a semiclassical way. For example, various estimates that are used in the proofs of the asymptotic completeness by the Enss method (see e.g. [E1, 2, 3, Pe]) belong to this category.

Another class of estimates that describe propagation of observables is related to the concept of the Kato-smoothness. We say that an operator B is locally H-smooth on the interval Δ if and only if the estimate

$$\int_{-\infty}^{\infty} \| Be^{iHt}\phi \|^2 dt < \infty$$

is satisfied for any vector ϕ that belongs to the range of the spectral projection of H onto Δ. (In the sequel we will just say "H-smooth" instead of "locally H-smooth".) This concept has been introduced by Kato in [Ka1, 2]. It has been used to prove various properties of Schrödinger operators such as the asymptotic completeness and the absence of the singular continuous spectrum. Let us name for instance the following references which used the H-smoothness (sometimes in a disguised form) [Pu, La1, 2, 3, 4, Ar, RS4, IoOC, Ha, HaPe, MS, Sig, SigSof1, De1].

The problem of finding H-smooth operators is especially interesting and nontrivial in the case of N-body Schrödinger operators. First of all, it can be shown that in this case, under quite mild conditions on the potentials, the operator

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