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# Large Field Renormalization. I. The Basic Step of the $\mathbb{R}$ Operation

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**Abstract.** We construct the renormalization operation of the expressions connected with the large field regions. This operation, denoted by $\mathbb{R}$, removes the main obstacle to prove the ultraviolet stability of four-dimensional gauge field theories. The proof will be completed in the second part of this paper.

## O. Introduction

Let us repeat briefly why it is necessary to renormalize the large field expressions, and what is a general structure of the operation $\mathbb{R}$. Consider a large plaquette variable in the first step. The restrictions on these variables are the same as in [16] (this refers to References in the paper [I]), so we have $|U(\partial p)-1|\geqq g_0 p_0(g_0)$ for a plaquette $p\in T_1$, where $p_0(g_0)=A_0(\log g_0^{-2})^{p_0}$ with a positive integer $p_0$. The term in the Wilson action, corresponding to the plaquette $p$, gives the estimate

$$\exp\left[-\frac{1}{g_0^2}[1-\operatorname{Retr} U(\partial p)]\right]\leqq \exp(-p_0(g_0))=g_0^{A_0(\log g_0^{-2})^{p_0-1}}. \tag{0.1}$$

For $d<4$ we have $g_0=g\varepsilon^{1/2(4-d)}$, and the bound above can be estimated by an arbitrarily large power of $\varepsilon$. This is enough to control expressions arising in the large field regions surrounding the plaquette $p$ for all steps of the procedure, i.e., until we reach the unit lattice. For $d=4$ the bare coupling constant behaves asymptotically as $(a+b\log\varepsilon^{-1})^{-1/2}$, for $\varepsilon\to 0$, with some positive constants $a,b$, hence the bound does not give any positive power of $\varepsilon$. It is still small for $\varepsilon$ small, and it controls a large number of steps, but this number is a small fraction of the total number of steps. Thus, for some large field regions there is a difficulty in continuing the procedure of [16], the small factor arising from large fields in this region does not control further steps. In such situations we have to change the procedure in order to improve the small factor, i.e., we have to be able to renormalize the expression corresponding to the large field region. There are several possible

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