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Comment

Redundancy of Conditions for a Virasoro Algebra*

Jack L. Uretsky

High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA
Department of Natural Sciences, College of DuPage, Glen Ellyn, IL 60137, USA

Abstract. I show that the Fairlie, Nuyts, Zachos construction of Virasoro algebra contains redundant conditions.

Fairlie et al. [1] construct a Virasoro algebra from two starting generators and eight conditions on the commutators. I show that the eight conditions are not independent.

The authors of [1] start with two generators called L_3 and L_{-2} and the following definitions:

$$\text{D1} \quad 5L_1 = [L_3, L_{-2}],$$

$$\text{D1} \quad 3L_{-1} = [L_1, L_{-2}],$$

$$\text{D1} \quad 2L_0 = [L_1, L_{-1}],$$

$$\text{D1} \quad 4L_2 = [L_2, L_{-1}],$$

$$\text{D1} \quad (n-1)L_{n+1} = [L_n, L_1] \qquad n \geqq 3,$$

$$\text{D1} \quad (n+1)L_{n-1} = [L_n, L_{-1}] \qquad n \leqq -2.$$

The authors then impose 8 conditions. I shall limit my discussion to positive values of the index n. The conditions that will be of interest are, then,

$$\text{C1} \quad [L_3, L_0] = 3L_3 \qquad \text{(Cond. 1 of ref. [1])},$$

$$\text{C2} \quad [L_0, L_{-2}] = 2L_{-2} \qquad \text{(Cond. 2 of ref. [1])},$$

$$\text{C3} \quad [L_2, L_{-2}] = 4L_0 + 6c \qquad \text{(Cond. 4 of ref. [1])},$$

$$\text{C4} \quad [L_2, L_1] = L_3 \qquad \text{(Cond. 3 of ref. [1])},$$

$$\text{C5} \quad [L_3, L_2] = L_5 \qquad \text{(Cond. 5 of ref. [1])},$$

$$\text{C6} \quad [L_5, L_2] = L_5 \qquad \text{(Cond. 5 of ref. [1])}.$$

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