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A One-Dimensional N Fermion Problem with Factorized S Matrix

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Abstract. A one dimensional N Fermion problem with attractive or repulsive δ function interaction is solved by Bethe's hypothesis. The S matrix factorizes and is explicitly given.

1. Introduction

We report a one dimensional N Fermion problem for which the S matrix is completely solved. The solution depends on extensive uses of the Yang-Baxter equation [1]. The corresponding problem for Boltzmann statistics was solved [2] in 1968, but specialization to Fermi statistics is algebraically involved and was never done. Here we approach the problem directly without going through the Boltzmann case.

The Hamiltonian for the problem is

$$H=-\sum_i \frac{\partial^2}{\partial x_i^2}+2c\sum_{i<j}\delta(x_i-x_j), \qquad (i,j=1,2,\dots,N), \tag{1}$$

where $c=$ real. Each particle has m "spin" states designated by $s_1, s_2 \dots s_N$ where

$$1 \leqq s_i \leqq m. \tag{2}$$

The Schrödinger equation is $H\psi=E\psi$, where

$$\psi=m^N\times 1, \qquad \text{(column)}. \tag{3}$$

For the Fermion problem we are only interested in wave functions ψ that are antisymmetrical with respect to the interchange:

$$Q^{ij}:(x_i,s_i)\leftrightarrow(x_j,s_j). \tag{4}$$

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