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Yang–Baxter Equations and Intermediate Long Wave Hierarchies

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Abstract. In the framework of the GN manifold approach, the algebraic structures of two representations of the ILW hierarchy are investigated. It is shown that this hierarchy can be obtained by different realizations of two abstract structures, strictly related with the Yang–Baxter equations. A new reduction theorem for the second representation of ILW is also formulated.

Introduction

It is well-known that the differential non-linear evolution equations (DNEE) in $(1+1)$ dimensions integrable by the Inverse Spectral Transform are bihamiltonian, with an infinite hierarchy of constants of the motions in involution [1]. More recently, this peculiar property has been pointed out also for many DNEE's in $(2+1)$ dimensions ([1, 2, 3] and references therein).

On the other hand, there are some integro-differential evolution equations (INEE) in $(1+1)$ dimensions, such as the Benjamin–Ono (BO) [4] and the Intermediate Long Wave equation (ILW) [5, 6, 7], which algebraic and analytical features quite similar to those of DNEE's in $(2+1)$ dimensions. In fact, they have been shown to be bihamiltonian [8, 9] only in the framework of the so-called extended formalism, which was introduced just for DNEE's in $(2+1)$ dimensions.

In this paper we investigate the algebraic structure of the ILW hierarchy and its deep relation with the Yang–Baxter equations [10, 11]. We make use of the GN *manifold* approach [3]: in this framework, by introducing a recursion (Nijenhuis) operator N and a symmetry group G of N one can construct the most part of the integrable bihamiltonian DNEE's in $(2+1)$ dimensions [12, 13]. The evolution equations are explicitly obtained by a suitable reduction of the *Lenard bicomplex*, i.e. a two-indices family of vector fields in involution making a Kac–Moody type algebra. Our first result is to show that the so-called *two* representations [9] of the ILW hierarchy correspond, roughly speaking, to particular solutions of *two* remarkable equations, the modified Yang–Baxter (mYB) equation and the Yang–Baxter (YB) equation. Indeed, we are able to construct two classes of GN manifolds and Lenard bicomplexes for any solution of these equations. Then, by choosing two particular solutions, we specialize the two structures and prove two reduction