

# On the Construction of Monopoles for the Classical Groups\*

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**Abstract.** For  $G$  a classical group, an equivalence is exhibited between:

- A)  $G$  monopoles over  $\mathbb{R}^3$ , with maximal symmetry breaking at infinity,
- B) families of  $(\text{rank}(G))$  algebraic curves in  $T\mathbb{P}_1$ , along with divisors on those curves, satisfying certain constraints,
- C) solutions of Nahm’s equations over  $(\text{rank}(G))$  intervals, satisfying the appropriate boundary conditions.

A) and B) are linked by twistor techniques, B) and C) via the Krichever method for solving non-linear differential equations, and A) and C) via the ADHMN construction, providing a unified picture of techniques for solution. Amongst other things, an asymptotic formula for the Higgs field of the monopole is computed.

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