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Uniqueness of Gibbs States for General $P(\varphi)_2$-Weak Coupling Models by Cluster Expansion

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Abstract. We consider quantum fields with weak coupling in two space-time dimensions. We prove that the set of their ultraregular Gibbs states consists of only one point and this point is an extremal Gibbs state.

1. Introduction

In this paper we prove the uniqueness of Gibbs states for general $P(\varphi)_2$-weak coupling models. This extends to the case of the $P(\varphi)_2$-model results proven before for weak trigonometric interactions [AHK1] and exponential interactions [Ze1, Gie1].

The method we use is the method of cluster expansion [GJS1, 2]. In a companion paper [AHKZ] we used other methods (essentially FKG-order) to yield a uniqueness result and the global Markov property for the φ_2^4-models. Weak coupling $P(\varphi)_2$-models have been constructed by Glimm et al. [GJS1, 2], see also [GlJa], using their method of cluster expansion, starting from the models given by an interaction confined to a bounded space-time region.

In analogy with classical statistical mechanics, see [Do, LaRu] and also e.g. [Pr] one can define Gibbs states associated with quantum fields given in a bounded space-time region. This has been first discussed by Guerra et al. [GRS1, 2], see also [Si] and pursued e.g. in [FrSi, DoMi, DoPe, AHK1]. Roughly speaking, the construction of Gibbs states corresponds to Kolmogorov's construction of Markov processes from Markov kernels ("local specification"). For these general connections see [Fö2] (who also discusses the relations with Martin-Dynkin's boundary). The work on Gibbs states and their local specifications from a potential theoretical point of view, applied to the study of quantum fields, has been pursued in [AHK1, Gie1, 2, Ze1–5, Rö1–3, RöZ]. The structure of Gibbs states is rather well understood in classical statistical mechanics, both in specific models (like in 2-dimensional Ising ferromagnets, where one has a complete structure theory by work of [Aiz, Hig], see also [Me]) and in general models (Pirogov-Sinai theory [Sin]; this theory uses preceding work by Dobrushin, Minlos-Sinai, Gercik).