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A Connection Between the Einstein and Yang-Mills Equations

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Abstract. It is our purpose here to show an unusual relationship between the Einstein equations and the Yang-Mills equations. We give a correspondence between solutions of the self-dual Einstein vacuum equations and the self-dual Yang-Mills equations with a special choice of gauge group. The extension of the argument to the full Yang-Mills equations yields Einstein's unifield equations. We try to incorporate the full Einstein vacuum equations, but the approach is incomplete. We first consider Yang-Mills theory for an arbitrary Lie-algebra with the condition that the connection 1-form and curvature are constant on Minkowski space. This leads to a set of algebraic equations on the connection components. We then specialize the Lie-algebra to be the (infinite dimensional) Lie-algebra of a group of diffeomorphisms of some manifold. The algebraic equations then become differential equations for four vector fields on the manifold on which the diffeomorphisms act. In the self-dual case, if we choose the connection components from the Lie-algebra of the volume preserving 4-dimensional diffeomorphism group, the resulting equations are the same as those obtained by Ashtekar, Jacobsen and Smolin, in their remarkable simplification of the self-dual Einstein vacuum equations. (An alternative derivation of the same equations begins with the self-dual Yang-Mills connection now depending only on the time, then choosing the Lie algebra as that of the volume preserving 3-dimensional diffeomorphisms.) When the reduced full Yang-Mills equations are used in the same context, we get Einstein's equations for his unified theory based on absolute parallelism. To incorporate the full Einstein *vacuum* equations we use as the Lie group the semi-direct product of the diffeomorphism group of a 4-dimensional manifold with the group of frame rotations of an $SO(1,3)$ bundle over the 4-manifold. This last approach, however, yields equations more general than the vacuum equations.

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