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# The Finite Toda Lattices

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**Abstract.** Connection is established between one-dimensional Toda lattices, constructed on the basis of the systems of simple roots of classical and affine Lie algebras, and other integrable systems of interacting particles. That connection allows us to find new lattices differing from the known ones by the interaction of particles near the ends. Some of the new lattices admit non-Abelian generalizations.

## 1. Introduction

The study of the propagation of waves in infinite one-dimensional lattices made it possible to derive exact analytic solutions to an infinite system of nonlinear differential equations describing the exponential interaction between the nearest neighbouring particles [1]. Once Henon, Flaschka [2] and Moser [3] have shown complete integrability of finite-dimensional Hamiltonian systems of that type (nonperiodic and periodic Toda lattices), a number of papers have appeared devoted to the investigation of their properties. Kostant [6], Olshanetsky and Perelomov [7] have established the connection between nonperiodic lattices and classical Lie algebras. Hamiltonians of those lattices may be constructed with the use of systems of simple roots $\{p\}$ of classical Lie algebras $\{\mathfrak{G}\}$,

$$H=\sum_{j=1}^{n}\frac{p_j^2}{2}+V_{\mathfrak{G}}, \qquad U_{\mathfrak{G}}=\sum_{\alpha\in\{p\}}\exp(\alpha q), \tag{1}$$

where $\alpha$ are root vectors, $p_j$ and $q_j$ are respectively momenta and coordinates of particles. For the algebras $\mathscr{A}_n$, $\mathscr{B}_n$, $\mathscr{C}_n$, and $\mathscr{D}_n$ the potentials $V_{\mathfrak{G}}$ are of the form

$$\begin{aligned} V_{\mathscr{A}_n}&=\sum_{j=1}^{n-1}\exp(q_j-q_{j+1}), \qquad U_{\mathscr{B}_n}=V_{\mathscr{A}_n}+\exp(q_n),\\ V_{\mathscr{C}_n}&=V_{\mathscr{A}_n}+\exp(2q_n), \qquad V_{\mathscr{D}_n}=U_{\mathscr{A}_n}+\exp(q_{n-1}+q_n). \end{aligned} \tag{2}$$