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New Bosonization and Conformal Field Theory over Z

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Dedicated to Professor M. Sato on his sixtieth birthday

Abstract. New formulation of bosonization is given so that it is defined over the ring **Z** of integers. The charge zero sector of the new boson Fock space is the completion of the coordinate ring of the universal Witt scheme. By using new bosonization, conformal field theory of free fermions over **Z** is given.

Introduction

String theory and conformal field theory have a deep connection with arithmetic geometry. (See for example, [ABMNV, B, BK, BM, FS] and references therein.) There are several attempts to generalize the theories in arithmetic directions: p-adic strings [FO], [V], adelic strings [FW, MA2], arithmetic bosonic strings [S, U], modular geometry of string theory and conformal field theory [F], conformal field theory over an arbitrary field which is related to automorphic representation [W]. (See also [MA1]. These papers are mainly based on the arithmetic properties of partition functions and correlation functions.

In the present paper we choose another approach to arithmetization of conformal field theory. Namely, conformal field theory of free fermions shall be realized arithmetico-geometrically so that the theory can be formulated over any commutative ring A with unity. If the ring A is the complex numbers $\mathbf{C}$, we have the usual conformal field theory.

Conformal field theory of free fermions on compact Riemann surfaces has a deep connection with geometry of the moduli space $\mathcal{M}_g$ of compact Riemann surfaces of genus g (cf. [ABMNV, AGR, BMS, BS, EO, IMO, and KNTY]). Especially, the determinant line bundle $\lambda_{1/2}$ of spin bundles plays an essential role in the theory. The moduli space $\mathcal{M}_g$ is an algebraic variety defined over the ring of integers $\mathbf{Z}$ and $\lambda_{1/2}$ is a line bundle on the moduli space $\mathcal{M}_{g,4}$ of level 4 structure defined over $\mathbf{Z}[\frac{1}{2}]$. Therefore, it is natural to ask whether the theory can be formulated over the integers $\mathbf{Z}$ or at least over $\mathbf{Z}[\frac{1}{2}]$. The main purpose of the

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