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Structure of Basic Lie Superalgebras and of their Affine Extensions

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Abstract. We generalize to the case of superalgebras several properties of simple Lie algebras involving the use of Dynkin diagrams. If to a simple Lie algebra can be associated one Dynkin diagram, it is a finite set of non-equivalent ones which can be constructed for a basic superalgebra (or B.S.A.). The knowledge of these diagrams, which can be obtained for each B.S.A. in a systematic way, allows us to deduce the regular subsuperalgebras of a B.S.A. The symmetries of the Dynkin diagrams are related to outer automorphisms of B.S.A. and lead to some singular subsuperalgebras. Finally we consider the extended Dynkin diagrams in order to classify the affine B.S.A. and use their symmetries to construct the twisted basic superalgebras.

1. Introduction

In his classification of simple Lie superalgebras, Kac [1–3] distinguishes two general families: the classical Lie superalgebras in which the representation of the even subalgebra on the odd part is completely reducible and the Cartan type superalgebras in which such a property is not valid. Among the classical superalgebras, one naturally separates the "strange" series $P(n)$ and $Q(n)$ from the basic or contragredient superalgebras which include the $A(m,n)$ unitary series, the $B(m,n)$, $C(n+1)$, and $D(m,n)$ orthosymplectic series and the exceptional superalgebras $F(4)$ and $G(3)$ as well as $D(2,1;\alpha)$ – these last ones being a deformation of $D(2,1)$. These basic superalgebras – up to now denoted by B.S.A. – are in some extent very close to the usual simple Lie algebras. For example, they can be studied with the help of Cartan matrices and Dynkin diagrams. However a fundamental difference with the Lie algebras occurs at this level because of the unavoidable presence in the simple root systems of odd – or fermionic – roots together with even – or bosonic – ones. Indeed for each simple Lie algebra $\mathscr{A}$, there is only one simple root system, up to a transformation of the Weyl group $W(\mathscr{A})$. In a B.S.A. several unequivalent simple root systems, that is systems which cannot be related one to each other by a Weyl transformation, can be in general defined,