Commun. Math. Phys. 445–455 (1989)

Communications in Mathematical Physics

Attractors via Random Perturbations*

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Abstract. We discuss conditions which ensure that weak limits of invariant measures of small random perturbations of dynamical systems have their supports on attractors.

1. Introduction

Let $F:M\to M$ be a continuous map of a metric space M. Suppose that $\{Q_x^\varepsilon, x\in M, \varepsilon>0\}$ is a family of probability distributions on M such that for any $\delta>0$ and each bounded continuous function g,

$$\limsup_{\varepsilon\to 0\, x\in M} |\int g(y)Q_x^\varepsilon(dy)-g(x)|=0. \tag{1.1}$$

Then the Markov chains X_n^ε with transition probabilities $p^\varepsilon(x,\Gamma)=P\{X_{n+1}^\varepsilon\in\Gamma|X_n^\varepsilon=x\}=Q_{Fx}^\varepsilon(\Gamma)$ are called random perturbations of iterates of the transformation F. One may be interested in the asymptotic behavior as $\varepsilon\to 0$ of invariant measures μ^ε of X_n^ε, i.e. the measures satisfying

$$\mu^\varepsilon(\cdot)=\int d\mu^\varepsilon(x)p^\varepsilon(x,\cdot). \tag{1.2}$$

Under (1.1) all weak limits μ of measures satisfying (1.2) are known to be invariant measures of F (see, for instance [K1], Sect. 1.1), i.e. $\mu(F^{-1}\Gamma)=\mu(\Gamma)$ for any Borel $\Gamma\subset M$.

In physical applications one may think on measures μ obtained as limits of μ^ε as more stable to random perturbations and so having more physical sense than other invariant measures. Usually physically relevant systems are considered near stable invariant sets, i.e. attractors and so their statistical behavior is thought to be described by invariant measures which sit on attractors. Thus it is important to specify conditions on random perturbations which ensure that weak limits of their invariant measures will sit on attractors. This will be the main issue of this

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* Talk presented at Durham Symposium on Dynamical Systems, July 1988. Partially supported by the U.S Army Research Office through the Mathematical Sciences Institute of Cornell University