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Asymptotics of Varadhan-Type and the Gibbs Variational Principle

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Abstract. For a large class of quantum models of mean-field type the thermodynamic limit of the free energy density is proved to be given by the Gibbs variational principle. The latter is shown to be equivalent to a non-commutative version of Varadhan's asymptotic formula.

1. Introduction

Varadhan developed a general theory of the asymptotics of integrals for measures satisfying a large deviation principle [19, 9]. An application of this theory generalizes results of Cramér on the rate of convergence in the weak law of large numbers [8]. Let $\xi_1, \xi_2, \ldots$ be a sequence of independent, identically distributed, random variables, and let μ_n be the distribution of the average

$$\bar{x}_n = (\xi_1 + \xi_2 + \ldots + \xi_n)/n .$$

Varadhan's result concerns the asymptotic behaviour of the measures v_n given by

$$dv_n(u) = \exp(nf(u))d\mu_n(u) .$$

for a continuous function f, and says that

$$\lim_{n \to \infty} n^{-1} \log v_n(R) = \sup\{f(u) - I(u) : u \in R\}, \tag{1}$$

where the rate-function I is determined by the distribution of ξ_1 [20, Sect. 3].

Here we consider the non-commutative analogue of (1); the random variables become self-adjoint operators in an operator algebra $\mathscr{A}$. To fix the ideas, let $\mathbf{M}$ be the algebra of all complex $m \times m$ matrices and

$$\mathscr{A} = \bigotimes_{i \in N} \mathbf{M}^i ,$$

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