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On the Existence of Equilibrium States in Local Quantum Field Theory

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Dedicated to E. H. Wichmann on the occasion of his 60th birthday

Abstract. It is shown that any local quantum field theory admits thermodynamical equilibrium states (KMS-states) for all positive temperatures provided it satisfies a "nuclearity condition," proposed by Wichmann and one of the authors, which restricts the admissible number of local degrees of freedom.

1. Introduction

Although the requirement of a reasonable thermodynamical interpretation is an indispensible condition on any fundamental physical theory it has not found an expression in the generally accepted postulates of local quantum field theory [1, 2]. There may be two reasons for this omission: first, quantum field theory is regarded primarily as a framework for the description of elementary systems (particles), even though its thermodynamical aspects are receiving increasing attention in recent years [3, 4]. And second, there is the common belief that the thermodynamical features of a theory can be derived from its microscopic properties by applying the rules of statistical mechanics. It seems therefore unnecessary to amend the basic postulates regulating these microscopic properties by further conditions.

Taking this pragmatic view one misses, however, the point that the rules of statistical mechanics are not always applicable: there exist quantum field theories which do not admit any thermodynamical equilibrium states [5]. Conversely, presuming a decent thermodynamical behaviour, one can establish interesting structural properties of the underlying elementary systems [6]. Thus a closer examination of the relation between the framework of quantum field theory and of quantum statistical mechanics seems to be of interest.

As a step in this direction we establish in the present article the existence of thermodynamical equilibrium states (KMS-states [7, 8]) for all positive temperatures in quantum field theories satisfying a nuclearity condition proposed in [6]. This crucial input restricts the number of local degrees of freedom of a theory in a physically sensible manner. It was argued in [5, 6] that the nuclearity condition