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Quantum Field Theories of Vortices and Anyons

J. Fröhlich[1] and P.A. Marchetti[2]

[1] Theoretical Physics, ETH-Hönggerberg, CH-8093 Zürich, Switzerland

[2] Dipartimento di Fisica, Universitá di Padova, I.N.F.N. Sez. di Padova, I-35131 Padova

Abstract. We develop the quantization of topological solitons (vortices) in three-dimensional quantum field theory, in terms of the Euclidean region functional integral. We analyze in some detail the vortices of the abelian Higgs model. If a Chern–Simons term is added to the action, the vortices turn out to be "anyons," i.e. particles with arbitrary real spin and intermediate (Θ) statistics. Localization properties of the interpolating field, scattering theory and spin-statistics connection of anyons are discussed. Such analysis might be relevant in connection with the fractional quantum Hall effect and two-dimensional models of High T_c superconductors.

1. Introduction

In this paper we consider the quantum theory of vortices in three space-time dimensions. Following the strategy of [1–2], we construct Euclidean Green functions of local order fields and vortex fields in terms of Euclidean region functional integrals. In these correlation functions the basic Euclidean fields of the theory are distributional sections of some non-trivial bundle over punctured 3-d Euclidean space-time, with those space-time points deleted where a vortex field is inserted.

We discuss, in detail, the non-compact abelian Higgs model, but the main strategy applies to a large class of models with vortices. The quantum vortex fields of these models can be localized in bounded regions and satisfy the dual algebra with the Wilson loop operators. In the Higgs phase, vortex fields of unit vorticity couple the vacuum to a massive stable one particle state; [for a rigorous proof on the lattice see [1]]. If a Chern–Simons term is added to the action of the model, then vortex fields still exist, carry a fractional electric charge proportional to the coefficient of the Chern–Simons term and cannot be localized in bounded regions. According to a general analysis of Buchholz and Fredenhagen [3], the electrically charged vortex fields can be localized in space-like cones in 3-d Minkowski space-time. Those with unit vorticity couple the vacuum to a massive stable one-particle state; with an "extended particle" structure.

Such particles are called "anyons", according to a terminology introduced by