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Global Validity of the Boltzmann Equation for Two- and Three-Dimensional Rare Gas in Vacuum: Erratum and Improved Result

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Abstract. We point out an error in our earlier papers [1] and [2] and present a more direct and natural proof which, although based on the same physical ideas of the previous ones, saves and actually improves the validity results for the Boltzmann equation given in [1] and [2].

1. The Error

We refer to the numbers from [1] as [1, (1.1)] say. The mistake in [1] is in [1, (3.8)]. The right conclusion of the arguments [1, (3.5–7)] leading to [1, (3.8)] is

$$I(\phi_t^d X) = I(\phi_t X) + \sum_{i=1}^{k} (t-t_i)(y_i' - y_i)(p_i' - u_i'), \tag{1.1}$$

with the original meaning of the symbols. Therefore, the lower bound [1, (3.3)] $I(\phi_t^d X) \geqq I(\phi_t X)$ remains correct, but the upper bound [1, (3.4)] is wrong. In Sect. 2, we show how the BBGKY-hierarchy can be controlled in a different, actually more efficient way.

By a slight generalization of the arguments leading to (1.1) it is easy to prove the following inequality (Lemma 1 below) showing that the interacting flow is more dispersive than the free one. This property enables us to control the solutions to the BBGKY and the Boltzmann hierarchies by one estimate along the lines followed in [1].

Lemma 1.

$$I(\phi_s \phi_t^d X) \geqq I(\phi_{s+t} X) \tag{1.2}$$

for all $s, t \geqq 0$ *or* $s, t \leqq 0$.

2. Estimate of the BBGKY Hierarchy

The application for which we needed [1, (3.8)] was to bound (uniformly in the diameter $d > 0$) the solution of the BBGKY hierarchy given by Lanford's