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On the Cauchy Problem for the Discrete Boltzmann Equation with Initial Values in $L_1^+(\mathbb{R})$

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Abstract. We prove a global existence theorem for a discrete velocity model of the Boltzmann equation when the initial values $\varphi_i(x)$ have finite entropy and, for some constant $\alpha > 0$, $(1+|x|^\alpha)\varphi_i(x) \in L_1^+(\mathbb{R})$.

1. Introduction

The discrete kinetic theory is concerned with the analysis of systems of gas particles with a finite set of selected velocities, and provides a useful substitute for the Boltzmann equation, in terms of a system of semilinear hyperbolic equations. This system defines the space-time evolution of the number of densities associated with every chosen velocity. Generally the discrete kinetic theory only takes into account the binary collisions.

These models, known as discrete velocity models of the Boltzmann equation, have been studied for some time now, but were introduced by Maxwell.

The general model is written in the form:

$$\frac{\partial f_i}{\partial t} + \vec{v}_i \cdot \nabla_x f_i = G_i(\underset{\sim}{f}, \underset{\sim}{f}) - f_i L_i(\underset{\sim}{f}) \qquad i = 1, 2, \ldots, r, \tag{1.1}$$

where $\vec{v}_i$, $i = 1, 2, \ldots, r$ is the set of the admissible velocities, and $\underset{\sim}{f}(\vec{x}, t) = \{f_1(\vec{x}, t), f_2(\vec{x}, t), \ldots, f_r(\vec{x}, t)\}$ is the r-component vector whose i^{th} component represents the density of the particles with velocity $\vec{v}_i$ is the position $\vec{x}$ at the time t. Both $\vec{x}$ and $\vec{v}$ are referred to an inertial reference frame S with unit vectors $\vec{i}, \vec{j}, \vec{k}$.

In the system (1.1) the gain term G_i and the loss term L_i are defined through the expressions:

$$\begin{cases} G_i(\underset{\sim}{f}, \underset{\sim}{f})(\vec{x}, t) = \dfrac{1}{2} \sum\limits_{j,k,m} A_{ij}^{km} f_k(\vec{x}, t) f_m(\vec{x}, t), \\ L_i(\underset{\sim}{f})(\vec{x}, t) = \dfrac{1}{2} \sum\limits_{j,k,m} A_{ij}^{km} f_j(\vec{x}, t). \end{cases} \tag{1.2}$$