Commun. Math. Phys. 121, 85–90 (1989)

Communications in Mathematical Physics

Why Instantons are Monopoles*

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Abstract. It is shown that instantons are hyperbolic monopoles for the loop group with non-maximal symmetry breaking at infinity.

1. Introduction

Monopoles in R^3 have been usually considered as time invariant instantons. We wish to turn this idea on its head and show that instantons are really monopoles on hyperbolic three space with structure group the loop group and with reduction at infinity to the subgroup of constant loops. This means that instantons are monopoles with non-maximal symmetry breaking.

This approach to instantons, while giving no new results, does shed light on the result of Atiyah that instantons are equivalent to rational maps from one dimensional complex projective space into the based loops which is the homogeneous space of the loop group divided by the constant loops. Atiyah's result then becomes another example of the general conjecture that monopoles, on R^3 or H^3 for a group K with reduction at infinity to a subgroup H are equivalent to rational maps of the two sphere into the homogeneous space K/H, or as Atiyah has described them, instantons for the corresponding two dimensional sigma model.

The status of this conjecture now is that on R^3 it has been proved for $SU(2)$ by Donaldson (1984) and for the other classical groups by Hurtubise (1988), all in the case of maximal symmetry breaking. For those hyperbolic monopoles which arise from invariant instantons (Atiyah 1984a) Atiyah has shown that the conjecture is also true, and lastly our observation along with Atiyah's results for instantons show that it is true for particular hyperbolic monopoles with loop group structure group and non-maximal symmetry breaking.

The correspondence between instantons and monopoles is explained in Sect. 2 and in Sect. 3 we show how the corresponding "twistor pictures" relate and how the spectral curve of an instanton arises. In the final section we make a conjecture motivated by this correspondence.

* Research supported in part by NSF grant DMS-8506130