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Convergence of the Fractional Step Lax–Friedrichs Scheme and Godunov Scheme for the Isentropic System of Gas Dynamics

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Abstract. A convergence theorem of the fractional step Lax–Friedrichs scheme and Godunov scheme for an inhomogeneous system of isentropic gas dynamics ($1 < \gamma \leqq 5/3$) is established by using the framework of compensated compactness. Meanwhile, a corresponding existence theorem of global solutions with large data containing the vacuum is obtained.

1. Introduction

We are concerned with the following Cauchy problem (1.1)–(1.2) for an inhomogeneous system of isentropic gas dynamics:

$$\begin{cases} \rho_t + (\rho u)_x = U(\rho, u, x, t), \\ (\rho u)_t + (\rho u^2 + p(\rho))_x = V(\rho, u, x, t), \end{cases} \tag{1.1}$$

$$(\rho, u)|_{t=0} = (\rho_0(x), u_0(x)). \tag{1.2}$$

Or

$$\begin{cases} v_t + f(v)_x = H(v, x, t), \\ v|_{t=0} = v_0(x, t), \end{cases} \tag{1.1$'$, 1.2$'$}$$

where $v = (\rho, m)^T, f(v) = (m, m^2/\rho + p(\rho))^T, H(v, x, t) = (U(\rho, m/\rho, x, t), V(\rho, m/\rho, x, t))^T$ and $m = \rho u, u_0(x)$ and $\rho_0(x) \geqq 0 (\not\equiv 0)$ are bounded measurable functions. For polytropic gas, $p(\rho) = k^2 \rho^\gamma$, where k is a constant and $\gamma > 1$ is the adiabatic exponent (for usually gases $1 < \gamma \leqq 5/3$).

System (1.1) is a model of gas dynamics of nonconservative form with source. For instance, $H(v, x, t) = (0, \alpha(x, t)\rho)^T$, where $\alpha(x, t)$ represents body force, usually gravity, acting on all the fluid in any volume. An essential feature of the system is a nonstrictly hyperbolicity, that is, a pair of wave speeds coalesce on the vacuum $\rho = 0$.

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