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Analytic Extrapolation in L^∞-Norm: An Alternative Approach to "QCD Sum Rules"

G. Auberson and G. Mennessier

Laboratoire de Physique Mathématique,* U.S.T.L., F-34060 Montpellier Cedex, France

Abstract. In view of physical applications (especially to "QCD Sum Rules"), the following problem, pertaining to analytic extrapolation techniques, is studied. We are considering "amplitudes," which are (real) analytic functions in the complex plane cut along $\Gamma = [s_0, \infty)$. A model $F_0(s)$ of the amplitude is given through the values of $F_0(s)$ on some interval $\gamma = [s_2, s_1]$ (with $s_1 < s_0$) and the values of its discontinuity on Γ. These values are approximate, and are supplemented by prescribed error channels, measured in L^∞-norm (both on Γ and γ). Investigating the compatibility between these data leads to an extremum problem which is solved up to a point where numerical methods can be implemented.

I. Introduction

Analytic extrapolation has been widely used in elementary particle theory to convey information between space-like and time-like domains of various amplitudes. In quantum chromodynamics (QCD) for instance, perturbation expansions, together with the inclusion of some non-perturbative effects, allow us to approximate the two-point functions of hadronic currents in the distant, space-like region in terms of a few parameters (the values of the so-called "condensates"). On the other hand, the discontinuity of these amplitudes in the time-like region is related to more directly measurable quantities. Although analyticity strongly correlates the values of the amplitudes in these two regions, the errors affecting both types of "data" make the correlation much looser. Any procedure aimed to build up acceptable amplitudes must take account of these errors in a reasonable way.

Several methods, generically called "QCD sum rules" have been devised to deal with this problem [1]. Most of them include the "theoretical errors" in the space-like region only at a qualitative level, and/or need (explicit or implicit) assumptions on the derivatives of the amplitudes. The application of fully controlled analytic extrapolation techniques should remedy these defects. As a matter of fact, a method of this sort has been already proposed, in which the error channels in

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* Unité Associée au CNRS n°040768