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# Non-Integrability of the 4-Vortex System: Analytical Proof

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**Abstract.** An analytical proof is given that the motion of $n$ point vortices in the plane is non-integrable for $n>3$. The basic geometric configuration, which models a situation often found experimentally, consists of two opposite strong vortices and two advected weak vortices. We use "Melnikov's method," as presented by Holmes and Marsden [Commun. Math. Phys. **82**, 523–544 (1982)]. The Melnikov integral is explicitly evaluated, by residues, in the limiting situation where one of the weak vortices is very close to one of the primaries.

## 1. Introduction

Our aim here is to present a completely analytical proof for the presence of horseshoes in the dynamics of 4 point vortices, using the Melnikov method as presented by Holmes and Marsden [HM]. The hydrodynamical background can be found in Aref [A1–A3]; for the mathematical relevance of this and other non-integrability problems, see Kozlov [Kz, Kz1]. The Dynamical Systems prerequisite here is the statement (with caveats, see Sect. 4): "the presence of transversal homoclinic points implies nonexistence of further (analytical) integrals of motion." Non-expert readers may find all the relevant material in Guckenheimer and Holmes [GH].

Our choice for a special geometrical setting consists of two strong, opposite vortices, and two weak, advected ones. Two-dimensional flows whose structure is dominated by strong, opposite couples are fairly common: Couder and Basdevant [CB] call those couples "Bachelor couples" in a remarkable double pun (they are very stable and a homage to Prof. Batchelor).

Ziglin [Z1, 1980], using another special configuration, gave a semi-analytical proof for the non-integrability: the Melnikov integral is evaluated numerically.

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