

Symplectic Fixed Points and Holomorphic Spheres

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Abstract. Let P be a symplectic manifold whose symplectic form, integrated over the spheres in P , is proportional to its first Chern class. On the loop space of P , we consider the variational theory of the symplectic action function perturbed by a Hamiltonian term. In particular, we associate to each isolated invariant set of its gradient flow an Abelian group with a cyclic grading. It is shown to have properties similar to the homology of the Conley index in locally compact spaces. As an application, we show that if the fixed point set of an exact diffeomorphism on P is nondegenerate, then it satisfies the Morse inequalities on P . We also discuss fixed point estimates for general exact diffeomorphisms.

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1. Introduction

1a. Hamiltonian Systems and the Arnold Conjecture. A symplectic structure on a smooth manifold P is a closed 2-form ω which for each $x \in P$ defines a nondegenerate bilinear form on $T_x P$. Due to the latter condition, ω provides a 1–1 correspondence