Commun. Math. Phys. 120, 525–527 (1989)

Communications in
Mathematical
Physics

Erratum

Convergence of Diffusion Waves of Solutions for Viscous Conservation Laws

I.-Liang Chern and Tai-Ping Liu

Commun. Math. Phys. **110**, 503–517 (1987)

In our paper the convergence rate should be lower due to a nonlinear interaction term we omitted. The nonlinear term (1.22) when rewritten on the last two lines of p. 511, its i^{th} component should be

$$N(a,b)=\frac{1}{2}\sum_{j\neq i} b_{ijj}a_j^2+\frac{1}{2}\sum_{j\neq k} b_{ijk}a_j a_k+\frac{1}{2}\sum_{j,k} b_{ijk}b_j b_k+l_i\cdot H(a,b),$$

$$b_{ijk}\equiv l_i(f''(0)\gamma_j,\gamma_k).$$

The first term on the right-hand side was missing in the original expression. It creates the interaction of i^{th} characteristic mode with other modes. This contributes to a lower rate of convergence of the solution to the diffusion waves. For instance the rate of L_1-convergence is around $t^{-1/4}$ instead of $t^{-1/2}$. The correct expression of (1.24) of the main result, Theorem 1.2 in [1] should be

$$\|D^l(u-\theta^*)\|_{L_p}(t)=0(1)\delta t^{-\left(\frac{l}{2}+\frac{3}{4}-\frac{1}{2p}-\sigma\right)}. \tag{1.24$'$}$$

This rate of convergence is in general optimal. We will present a simple example later to illustrate this. The rate of $t^{-\frac{1}{4}}$ for L_1-convergence is consistent with the inviscid theory. The same rate was obtained in [4] for convergence of solutions of hyperbolic conservation laws to N-waves. The L_2-result has also been obtained independently by Kawashima in [2]. The L_1-result for physical systems which are hyperbolic-parabolic has not been obtained.

To obtain this we follow the same technique as before and use the integration form of (1.19) through parametric methods. The missing term yields

$$\xi_i\equiv\int_0^t\int_{-\infty}^{\infty} G_i(x+y,t-s)\,\frac{1}{2}\sum_{j\neq i} b_{ijj}(\theta_j^2(y,s))_y\,dy\,ds,$$

where

$$G_i(t,t)\equiv\frac{1}{\sqrt{2\pi t}}\exp\left(-\frac{(x-\lambda_i t)^2}{2t}\right).$$